Tibor Pál, PhD candidateTibor PálPhD candidateTibor is a Ph.D. candidate in Statistics at the University of Salerno, focusing on time series models applied in macroeconomics and finance. His work is greatly motivated by the perception that risk, uncertainty, and unexpected events are inherent driving features of everyone's lives; thus, attitude towards these aspects is essential to one's life and economics. Hence, his primary interest is developing novel statistical approaches to capture unordinary episodes in economic activity and irregularities in the financial market driven by risk-related behaviors. Translating these elements into his life makes him keen to discover and live in unusual places where he can always bring some unique features into his daily life. He likes gastronomy, nature, and mountains, so traveling, cooking, and hiking are his favorite activities in his free time. See full profileCheck our editorial policyAnna Szczepanek, PhDAnna SzczepanekPhD, Jagiellonian University in Kraków, PolandAnna Szczepanek, PhD is a mathematician at the Faculty of Mathematics and Computer Science of the Jagiellonian University in Kraków, where she researches mathematical physics and applied mathematics. At Omni, Anna uses her knowledge and programming skills to create math and statistics calculators. In her free time, she enjoys hiking and reading. See full profileCheck our editorial policy and Jack Bowater140 people find this calculator helpfulTable of contentsUse the t-statistic calculator (t-value calculator or t test statistic calculator) to compute the t-value of a given dataset using its sample mean, population mean, standard deviation and sample size. Read further where we answer to the following questions: What is the t-statistic? How do I calculate t-statistic? What is the difference between T-score vs. Z-score? If you are also interested in F-test, check our F-statistic calculator.In statistics, the t-statistic, or t-value, is a measure that describes the relationship between a sample and its population. The t-statistic is central to the Student's t-test, which is a test for evaluating hypotheses about the population mean.More precisely, the t-statistic is used to determine whether to support or reject the null hypothesis. It is used in conjunction with the p-value, or critical value, which indicates the probability that your results could have happened by chance. It is comparable to the z-statistic, with the difference being that the t-statistic is applied for small sample sizes or unknown population standard deviations.You need to use the following t-statistic formula to calculate the t-value:t=x¯−μs/nt = \frac{\bar x - \mu}{s/\sqrt n}t=s/nx¯−μWhere: x¯\bar xx¯ - Sample mean; μ\muμ - Population mean; nnn - Sample size; and sss - Standard deviation of the sample. To compute the t-statistic, you need to provide the following four variables: Sample mean, x¯\bar xx¯; Population mean, μ\muμ; Sample size, nnn; and Sample standard deviation, sss. Alternatively, you can use the tool in reverse; for example, you can recover the sample mean from the t-statistic, provided you input all other values.Let's say you are a basketball player and your game score is 15 (x̄) on average over 36 (n) games, with a standard deviation of 6 (s). You know that an average basketball player scores 10 (μ). Should your performance be considered above average? Or are your scores due to luck? Finding the t-statistic and the probability value will give you some insight. More specifically, finding the t-statistic with the p-value will let you know if there is a significant difference between your mean and the population mean of everyone else. Applying the previously stated t-statistic formula, you can obtain the following equation. t=15−106/36=5t = \dfrac{15 - 10}{6 / \sqrt{36}} = 5t=6/3615−10=5 Now, we know that the t-statistic equals 5, but what does it mean? To gain more knowledge, you should compare this value with a particular threshold (or significance level), let's say 5 percent (α = 5%) of a Student-t distribution. Since the sample size is relatively large (n > 30) we can use the critical value of the standard normal distribution. The critical value of a 5% threshold in a standard normal distribution is 1.645. Since our t-statistic is above the critical value, we can say that you play better than the average. In fact, we have just performed a Student's t-test! Visit our dedicated t-test calculator to learn more.Both t-score and Z-score aim to make comparisons and decide on the dissimilarity between the sample and the population mean. The main difference between T-score vs. Z-score comes from the knowledge about the population standard deviation. For Z-score, we assume it is given, while for T-score you need to estimate it. In addition, T-score can be applied when you have a small sample size (less than 30 elements).To calculate t-statistic: Determine the sample mean (x̄, x bar), which is the arithmetic mean of your data set. Find the population mean (μ, mu). Compute the sample standard deviation (s) by taking the square root of the variance. To find the variance, if it is not given, take each value in the sample, subtract it from the sample mean, square the difference, sum them up, and divide by the sample size minus one. Calculate the t-statistic as (x̄ - μ) / (s / √n), where n denotes the sample size. The student t-test was devised by Gosset, who developed the connected statistical theory in 1908. At the time, Gosset worked at the Guinness Brewery in Dublin, which had an internal policy of forbidding employees from publishing to preclude potential loss of trade secrets. Gosset, however, found a loophole: he was writing under the pseudonym of 'Student'. As a consequence, the statistical student t distribution became known as student t rather than Gosset's t. So, next time you enjoying a pint of Guinness with your friend, you have a compelling story to share.Sample standard deviation (s)Did we solve your problem today?Check out 28 similar inference, regression, and statistical tests calculators Probability distribution This article is about the mathematics of Student's t-distribution. For its uses in statistics, see Student's t-test. Student's t Probability density function Cumulative distribution functionParameters ν > 0 {\displaystyle u >0} degrees of freedom (real, almost always a positive integer)Support x ∈ (− ∞ , ∞) {\displaystyle x\in (-\infty ,\infty)} PDF Γ (ν + 1 2) π ν Γ (ν 2) (1 + x 2 ν) − ν + 1 2 {\displaystyle {\frac {\Gamma \left({\frac {u +1}{2}}\right)}{{\sqrt {\pi u }}\,\Gamma \left({\frac {u }{2}}\right)}}\left(1+{\frac {x^{2}}{u }}\right)^{-{\frac {u +1}{2}}}} CDF 1 2 + x Γ (ν + 1 2) × 2 F 1 (1 2 , ν + 1 2 ; 3 2 ; − x 2 ν) π ν Γ (ν 2) , {\displaystyle {\begin{aligned}&{\frac {1}{2}}+x\Gamma \left({\frac {u +1}{2}}\right)\times \\&\quad {\frac {{}_{2}F_{1}\!\left({\frac {1}{2}},{\frac {u +1}{2}};{\frac {3}{2}};-{\frac {x^{2}}{u }}\right)}{{\sqrt {\pi u }}\,\Gamma \left({\frac {u }{2}}\right)}},\end{aligned}}} where 2 F 1 {\displaystyle {}_{2}F_{1}} is the hypergeometric functionMean 0 {\displaystyle 0} for ν > 1 , {\displaystyle u >1,} otherwise undefinedMedian 0 {\displaystyle 0} Mode 0 {\displaystyle 0} Variance ν ν − 2 {\displaystyle {\frac {u }{u -2}}} for ν > 2 , {\displaystyle u >2,} ∞ {\displaystyle \infty } for 1 < ν ≤ 2 , {\displaystyle 1 3 , {\displaystyle \ u >3\ ,} otherwise undefinedExcess kurtosis 6 ν − 4 {\displaystyle {\frac {6}{u -4}}} for ν > 4 , {\displaystyle u >4,} ∞ {\displaystyle \infty } for 2 < ν ≤ 4 , {\displaystyle 2 0 {\displaystyle u >0} , where K ν {\displaystyle K_{u }} is the modified Bessel function of the second kind[1]Expected shortfall μ + s ((ν + [T − 1 (1 − p)] 2) × τ (T − 1 (1 − p)) (ν − 1) (1 − p)) , {\displaystyle \mu +s\left({\frac {{\big (}u +[T^{-1}(1-p)]^{2}{\big)}\times \tau {\big (}T^{-1}(1-p){\big)}}{(u -1)(1-p)}}\right),} where T − 1 {\displaystyle T^{-1}} is the inverse standardized Student t CDF, and τ {\displaystyle \tau } is the standardized Student t PDF.[2] In probability theory and statistics, Student's t distribution (or simply the t distribution) t ν {\displaystyle t_{u }} is a continuous probability distribution that generalizes the standard normal distribution. Like the latter, it is symmetric around zero and bell-shaped. However, t ν {\displaystyle t_{u }} has heavier tails, and the amount of probability mass in the tails is controlled by the parameter ν {\displaystyle u } . For ν = 1 {\displaystyle u =1} the Student's t distribution t ν {\displaystyle t_{u }} becomes the standard Cauchy distribution, which has very "fat" tails; whereas for ν → ∞ {\displaystyle u \to \infty } it becomes the standard normal distribution N (0 , 1) , {\displaystyle {\mathcal {N}}(0,1),} which has very "thin" tails. The name "Student" is a pseudonym used by William Sealy Gosset in his scientific paper publications during his work at the Guinness Brewery in Dublin, Ireland. The Student's t distribution plays a role in a number of widely used statistical analyses, including Student's t-test for assessing the statistical significance of the difference between two sample means, the construction of confidence intervals for the difference between two population means, and in linear regression analysis. In the form of the location-scale t distribution ℓ s t (μ , τ 2 , ν) {\displaystyle \operatorname {\ell st} (\mu ,\tau ^{2},u)} it generalizes the normal distribution and also arises in the Bayesian analysis of data from a normal family as a compound distribution when marginalizing over the variance parameter. Student's t distribution has the probability density function (PDF) given by f (t) = Γ (ν + 1 2) π ν Γ (ν 2) (1 + t 2 ν) − (ν + 1) / 2 , {\displaystyle f(t)={\frac {\Gamma \left({\frac {u +1}{2}}\right)}{{\sqrt {\pi u }}\Gamma \left({\frac {u }{2}}\right)}}\left(1+{\frac {t^{2}}{u }}\right)^{-(u +1)/2},} where ν {\displaystyle u } is the number of degrees of freedom, and Γ {\displaystyle \Gamma } is the gamma function. This may also be written as f (t) = 1 ν B (1 2 , ν 2) (1 + t 2 ν) − (ν + 1) / 2 , {\displaystyle f(t)={\frac {1}{{\sqrt {u }}\,\mathrm {B} \left({\frac {1}{2}},{\frac {u }{2}}\right)}}\left(1+{\frac {t^{2}}{u }}\right)^{-(u +1)/2},} where B {\displaystyle \mathrm {B} } is the beta function. In particular for integer valued degrees of freedom ν {\displaystyle u } we have: For ν > 1 {\displaystyle u >1} and even, Γ (ν + 1 2) π ν Γ (ν 2) = 1 2 ν · (ν − 1) · (ν − 3) ⋯ 5 · 3 (ν − 2) · (ν − 4) ⋯ 4 · 2 . {\displaystyle {\frac {\Gamma \left({\frac {u +1}{2}}\right)}{{\sqrt {\pi u }}\,\Gamma \left({\frac {u }{2}}\right)}}={\frac {1}{2{\sqrt {u }}}}\cdot {\frac {(u -1)\cdot (u -3)\cdots 5\cdot 3}{(u -2)\cdot (u -4)\cdots 4\cdot 2}}.} For ν > 1 {\displaystyle u >1} and odd, Γ (ν + 1 2) π ν Γ (ν 2) = 1 π ν · (ν − 1) · (ν − 3) ⋯ 4 · 2 (ν − 2) · (ν − 4) ⋯ 5 · 3 . {\displaystyle {\frac {\Gamma \left({\frac {u +1}{2}}\right)}{{\sqrt {\pi u }}\,\Gamma \left({\frac {u }{2}}\right)}}={\frac {1}{\pi {\sqrt {u }}}}\cdot {\frac {(u -1)\cdot (u -3)\cdots 4\cdot 2}{(u -2)\cdot (u -4)\cdots 5\cdot 3}}.} The probability density function is symmetric, and its overall shape resembles the bell shape of a normally distributed variable with mean 0 and variance 1, except that it is a bit lower and wider. As the number of degrees of freedom grows, the t distribution approaches the normal distribution with mean 0 and variance 1. For this reason ν {\displaystyle {u }} is also known as the normality parameter.[3] The following images show the density of the t distribution for increasing values of ν . {\displaystyle u .} The normal distribution is shown as a blue line for comparison. Note that the t distribution (red line) becomes closer to the normal distribution as ν {\displaystyle u } increases. Density of the t distribution (red) for 1, 2, 3, 5, 10, and 30 degrees of freedom compared to the standard normal distribution (blue).Previous plots shown in green.1 degree of freedom2 degrees of freedom3 degrees of freedom5 degrees of freedom10 degrees of freedom30 degrees of freedom The cumulative distribution function (CDF) can be written in terms of I, the regularized incomplete beta function. For t > 0 , F (t) = ∫ − ∞ t f (u) d u = 1 − 1 2 I x (t) (ν 2 , 1 2) , {\displaystyle F(t)=\int _{-\infty }^{t}\ f(u)\ \operatorname {d} u~=~1-{\frac {1}{2}}I_{x(t)}\!\left({\frac {\ u \ }{2}},\ {\frac {\ 1\ }{2}}\right)\ ,} where x (t) = ν t 2 + ν . {\displaystyle x(t)={\frac {u }{\ t^{2}+u \ }}~.} Other values would be obtained by symmetry. An alternative formula, valid for t 2 < ν , {\displaystyle \ t^{2} 1 , {\displaystyle u >1\ ,} the raw moments of the t distribution are E { T k } = { 0 k odd , 0 < k < ν , 1 π Γ (ν 2) [Γ (k + 1 2) Γ (ν − k 2) ν k 2] k even , 0 < k < ν . {\displaystyle \operatorname {\mathbb {E} } \left\{\ T^{k}\ \right\}={\begin{cases}\quad 0&{\text{ odd }},\quad 04~.} Student's t-distribution with ν {\displaystyle u } degrees of freedom can be defined as the distribution of the random variable T with[5][6] T = Z V / ν = Z ν V , {\displaystyle T={\frac {Z}{\sqrt {V/u }}}=Z{\sqrt {\frac {u }{V}}},} where Z is a standard normal with expected value 0 and variance 1; V has a chi-squared distribution (χ2-distribution) with ν {\displaystyle u } degrees of freedom; Z and V are independent; A different distribution is defined as that of the random variable defined, for a given constant μ, by (Z + μ) ν V . {\displaystyle (Z+\mu){\sqrt {\frac {u }{V}}}.} This random variable has a noncentral t-distribution with noncentrality parameter μ. This distribution is important in studies of the power of Student's t-test. Suppose X1, ..., Xn are independent realizations of the normally-distributed, random variable X, which has an expected value μ and variance σ2. Let X ¯ n = 1 n (X 1 + ⋯ + X n) {\displaystyle {\overline {X}}_{n}={\frac {1}{n}}(X_{1}+\cdots +X_{n})} be the sample mean, and s 2 = 1 n − 1 ∑ i = 1 n (X i − X ¯ n) 2 {\displaystyle s^{2}={\frac {1}{n-1}}\sum _{i=1}^{n}\left(X_{i}-{\overline {X}}_{n}\right)^{2}} be an unbiased estimate of the variance from the sample. It can be shown that the random variable V = (n − 1) s 2 σ 2 {\displaystyle V=(n-1){\frac {s^{2}}{\sigma ^{2}}}} has a chi-squared distribution with ν = n − 1 {\displaystyle u =n-1} degrees of freedom (by Cochran's theorem).[7] It is readily shown that the quantity Z = (X ¯ n − μ) n σ {\displaystyle Z=\left({\overline {X}}_{n}-\mu \right){\frac {\sqrt {n}}{\sigma }}} is normally distributed with mean 0 and variance 1, since the sample mean X ¯ n {\displaystyle {\overline {X}}_{n}} is normally distributed with mean μ and variance σ2/n. Moreover, it is possible to show that these two random variables (the normally distributed one Z and the chi-squared-distributed one V) are independent. Consequently[clarification needed] the pivotal quantity T ≡ Z V / ν = (X ¯ n − μ) n s , {\textstyle T\equiv {\frac {Z}{\sqrt {V/u }}}=\left({\overline {X}}_{n}-\mu \right){\frac {\sqrt {n}}{s}},} which differs from Z in that the exact standard deviation σ is replaced by the sample standard error s, has a Student's t-distribution as defined above. Notice that the unknown population variance σ2 does not appear in T, since it was in both the numerator and the denominator, so it canceled. Gosset intuitively obtained the probability density function stated above, with ν {\displaystyle u } equal to n − 1, and Fisher proved it in 1925.[8] The distribution of the test statistic T depends on ν {\displaystyle u } , but not μ or σ; the lack of dependence on μ and σ is what makes the t-distribution important in both theory and practice. The t distribution arises as the sampling distribution of the t statistic. Below the one-sample t statistic is discussed, for the corresponding two-sample t statistic see Student's t-test. Let x 1 , … , x n ∼ N (μ , σ 2) {\displaystyle \ x_{1},\ldots ,x_{n}\sim {\mathcal {N}}(\mu ,\sigma ^{2})\ } be independent and identically distributed samples from a normal distribution with mean μ {\displaystyle \mu } and variance σ 2 . {\displaystyle \ \sigma ^{2}~.} The sample mean and unbiased sample variance are given by: x ¯ = x 1 + ⋯ + x n n , s 2 = 1 n − 1 ∑ i = 1 n (x i − x ¯) 2 . {\displaystyle {\begin{aligned}{\bar {x}}&={\frac {\ x_{1}+\cdots +x_{n}\ }{n}}\ ,\\[5pt]s^{2}&={\frac {1}{\ n-1\ }}\ \sum _{i=1}^{n}(x_{i}-{\bar {x}})^{2}~.\end{aligned}}} The resulting (one sample) t statistic is given by t = x ¯ − μ s / n ∼ t n − 1 . {\displaystyle t={\frac {{\bar {x}}-\mu }{\ s/{\sqrt {n\ }}\ }}\sim t_{n-1}~.} and is distributed according to a Student's t distribution with n − 1 {\displaystyle \ n-1\ } degrees of freedom. Thus for inference purposes the t statistic is a useful "pivotal quantity" in the case when the mean and variance (μ , σ 2) {\displaystyle (\mu ,\sigma ^{2})} are unknown population parameters, in the sense that the t statistic has then a probability distribution that depends on neither μ {\displaystyle \mu } nor σ 2 . {\displaystyle \ \sigma ^{2}~.} Instead of the unbiased estimate s 2 {\displaystyle \ s^{2}\ } we may also use the maximum likelihood estimate s M L 2 = 1 n ∑ i = 1 n (x i − x ¯) 2 {\displaystyle \ s_{\mathsf {ML}}^{2}={\frac {\ 1\ }{n}}\ \sum _{i=1}^{n}(x_{i}-{\bar {x}})^{2}\ } yielding the statistic t M L = x ¯ − μ s M L 2 / n = n n − 1 t . {\displaystyle \ t_{\mathsf {ML}}={\frac {{\bar {x}}-\mu }{\sqrt {s_{\mathsf {ML}}^{2}/n\ }}}={\sqrt {{\frac {n}{n-1}}\ }}\ t~.} This is distributed according to the location-scale t distribution: t M L ∼ ℓ s t (0 , τ 2 = n / (n − 1) , n − 1) . {\displaystyle t_{\mathsf {ML}}\sim \operatorname {\ell st} (0,\ \tau ^{2}=n/(n-1),\ n-1)~.} The location-scale t distribution results from compounding a Gaussian distribution (normal distribution) with mean μ {\displaystyle \ \mu \ } and unknown variance, with an inverse gamma distribution placed over the variance with parameters a = ν 2 {\displaystyle \ a={\frac {\ u \ }{2}}\ } and b = ν τ 2 2 . {\displaystyle b={\frac {\ u \ \tau ^{2}\ }{2}}~.} In other words, the random variable X is assumed to have a Gaussian distribution with an unknown variance distributed as inverse gamma, and then the variance is marginalized out (integrated out). Equivalently, this distribution results from compounding a Gaussian distribution with a scaled-inverse-chi-squared distribution with parameters ν {\displaystyle u } and τ 2 . {\displaystyle \ \tau ^{2}~.} The scaled-inverse-chi-squared distribution is exactly the same distribution as the inverse gamma distribution, but with a different parameterization, i.e. ν = 2 a , τ 2 = b a . {\displaystyle \ u =2\ a,\;{\tau }^{2}={\frac {\ b\ }{a}}~.} The reason for the usefulness of this characterization is that in Bayesian statistics the inverse gamma distribution is the conjugate prior distribution of the variance of a Gaussian distribution. As a result, the location-scale t distribution arises naturally in many Bayesian inference problems.[9] Student's t distribution is the maximum entropy probability distribution for a random variate X having a certain value of E { ln (ν + X 2) } {\displaystyle \ \operatorname {\mathbb {E} } \left\{\ \ln(u +X^{2})\ \right\}\ } .[10][clarification needed][better source needed] This follows immediately from the observation that the pdf can be written in exponential family form with ν + X 2 {\displaystyle u +X^{2}} as sufficient statistic. The function A(t | ν) is the integral of Student's probability density function, f(t) between -t and t, for t ≥ 0 . It thus gives the probability that a value of t less than that calculated from observed data would occur by chance. Therefore, the function A(t | ν) can be used when testing whether the difference between the means of two sets of data is statistically significant, by calculating the corresponding value of t and the probability of its occurrence if the two sets of data were drawn from the same population. This is used in a variety of situations, particularly in t tests. For the statistic t, with ν degrees of freedom, A(t | ν) is the probability that t would be less than the observed value if the two means were the same (provided that the smaller mean is subtracted from the larger, so that t ≥ 0). It can be easily calculated from the cumulative distribution function Fν(t) of the t distribution: A (t | ν) = F ν (t) − F ν (− t) = 1 − I ν ν + t 2 (ν 2 , 1 2) , {\displaystyle A(t\mid u)=F_{u }(t)-F_{u }(-t)=1-I_{\frac {u }{u +t^{2}}}\!\left({\frac {u }{2}},{\frac {1}{2}}\right),} where Ix(a, b) is the regularized incomplete beta function. For statistical hypothesis testing this function is used to construct the p-value. The noncentral t distribution generalizes the t distribution to include a noncentrality parameter. Unlike the nonstandardized t distributions, the noncentral distributions are not symmetric (the median is not the same as the mode). The discrete Student's t distribution is defined by its probability mass function at r being proportional to:[11] ∏ j = 1 k 1 (r + j + a) 2 + b 2 r = … , − 1 , 0 , 1 , … . {\displaystyle \prod _{j=1}^{k}{\frac {1}{(r+j+a)^{2}+b^{2}}}\quad \quad r=\ldots ,-1,0,1,\ldots ~.} Here a, b, and k are parameters. This distribution arises from the construction of a system of discrete distributions similar to that of the Pearson distributions for continuous distributions.[12] One can generate Student A(t | ν) samples by taking the ratio of variables from the normal distribution and the square-root of the χ² distribution. If we use instead of the normal distribution, e.g., the Irwin–Hall distribution, we obtain over-all a symmetric 4 parameter distribution, which includes the normal, the uniform, the triangular, the Student t and the Cauchy distribution. This is also more flexible than some other symmetric generalizations of the normal distribution. t distribution is an instance of ratio distributions. The square of a random variable distributed tn is distributed as Snedecor's F distribution F1,n. Student's t distribution generalizes to the three parameter location-scale t distribution ℓ s t (μ , τ 2 , ν) {\displaystyle \operatorname {\ell st} (\mu ,\ \tau ^{2},\ u)\ } by introducing a location parameter μ {\displaystyle \ \mu \ } and a scale parameter τ . {\displaystyle \ \tau ~.} With T ∼ t ν {\displaystyle \ T\sim t_{u }\ } and location-scale family transformation X = μ + τ T {\displaystyle \ X=\mu +\tau \ T\ } we get X ∼ ℓ s t (μ , τ 2 , ν) . {\displaystyle \ X\sim \operatorname {\ell st} (\mu ,\ \tau ^{2},\ u)~.} The resulting distribution is also called the non-standardized Student's t distribution. The location-scale t distribution has a density defined by:[13] p (x | ν , μ , τ) = Γ (ν + 1 2) Γ (ν 2) τ π ν (1 + 1 ν (x − μ τ) 2) − (ν + 1) / 2 {\displaystyle p(x\mid u ,\mu ,\tau)={\frac {\Gamma \left({\frac {u +1}{2}}\right)}{\Gamma \left({\frac {u }{2}}\right)\tau {\sqrt {\pi u }}}}\left(1+{\frac {1}{u }}\left({\frac {x-\mu }{\tau }}\right)^{2}\right)^{-(u +1)/2}} Equivalently, the density can be written in terms of τ 2 {\displaystyle \tau ^{2}} : p (x | ν , μ , τ 2) = Γ (ν + 1 2) Γ (ν 2) π ν τ 2 (1 + 1 ν (x − μ) 2 τ 2) − (ν + 1) / 2 {\displaystyle \ p(x\mid u ,\mu ,\tau ^{2})={\frac {\Gamma ({\frac {u +1}{2}})}{\Gamma ({\frac {u }{2}}){\sqrt {\pi u \tau ^{2}}}}}\left(1+{\frac {1}{u }}{\frac {(x-\mu)^{2}}{\tau ^{2}}}\right)^{-(u +1)/2}} Other properties of this version of the distribution are:[13] E { X } = μ for ν > 1 , var { X } = τ 2 ν ν − 2 for ν > 2 , mode { X } = μ . {\displaystyle {\begin{aligned}\operatorname {\mathbb {E} } \{\ X\ \}&=\mu &{\text{ for }}u >1\ ,\\\operatorname {var} \{\ X\ \}&=\tau ^{2}{\frac {u }{u -2}}&{\text{ for }}u >2\ ,\\\operatorname {mode} \{\ X\ \}&=\mu ~.\end{aligned}}} If X {\displaystyle \ X\ } follows a location-scale t distribution X ∼ ℓ s t (μ , τ 2 , ν) {\displaystyle \ X\sim \operatorname {\ell st} \left(\mu ,\ \tau ^{2},\ u \right)\ } then for ν → ∞ {\displaystyle \ u \rightarrow \infty \ } X {\displaystyle \ X\ } is normally distributed X ∼ N (μ , τ 2) {\displaystyle X\sim \mathrm {N} \left(\mu ,\tau ^{2}\right)} with mean μ {\displaystyle \mu } and variance τ 2 . {\displaystyle \ \tau ^{2}~.} The location-scale t distribution ℓ s t (μ , τ 2 , ν = 1) {\displaystyle \ \operatorname {\ell st} \left(\mu ,\ \tau ^{2},\ u =1\right)\ } with degree of freedom ν = 1 {\displaystyle u =1} is equivalent to the Cauchy distribution C a u (μ , τ) . {\displaystyle \mathrm {Cau} \left(\mu ,\tau \right)~.} The location-scale t distribution ℓ s t (μ = 0 , τ 2 = 1 , ν) {\displaystyle \operatorname {\ell st} \left(\mu =0,\ \tau ^{2}=1,\ u \right)\ } with μ = 0 {\displaystyle \mu =0} and τ 2 = 1 {\displaystyle \ \tau ^{2}=1\ } reduces to the Student's t distribution t ν . {\displaystyle \ t_{u }~.} Student's t distribution arises in a variety of statistical estimation problems where the goal is to estimate an unknown parameter, such as a mean value, in a setting where the data are observed with additive errors. If (as in nearly all practical statistical work) the population standard deviation of these errors is unknown and has to be estimated from the data, the t distribution is often used to account for the extra uncertainty that results from this estimation. In most such problems, if the standard deviation of the errors were known, a normal distribution would be used instead of the t distribution. Confidence intervals and hypothesis tests are two statistical procedures in which the quantiles of the sampling distribution of a particular statistic (e.g. the standard score) are required. In any situation where this statistic is a linear function of the data, divided by the usual estimate of the standard deviation, the resulting quantity can be rescaled and centered to follow Student's t distribution. Statistical analyses involving means, weighted means, and regression coefficients all lead to statistics having this form. Quite often, textbook problems will treat the population standard deviation as if it were known and thereby avoid the need to use the Student's t distribution. These problems are generally of two kinds: (1) those in which the sample size is so large that one may treat a data-based estimate of the variance as if it were certain, and (2) those that illustrate mathematical reasoning, in which the problem of estimating the standard deviation is temporarily ignored because that is not the point that the author or instructor is then explaining. A number of statistics can be shown to have t distributions for samples of moderate size under null hypotheses that are of interest, so that the t distribution forms the basis for significance tests. For example, the distribution of Spearman's rank correlation coefficient ρ, in the null case (zero correlation) is well approximated by the t distribution for sample sizes above about 20.[citation needed] Suppose the number A is so chosen that P { − A < T < A } = 0.9 , {\displaystyle \ \operatorname {\mathbb {P} } \left\{\ {-A}2,} ∞ {\displaystyle \infty } for 1 < ν ≤ 2 , {\displaystyle 1 3 , {\displaystyle \ u >3\ ,} otherwise undefinedExcess kurtosis 6 ν − 4 {\displaystyle {\frac {6}{u -4}}} for ν > 4 , {\displaystyle u >4,} ∞ {\displaystyle \infty } for 2 < ν ≤ 4 , {\displaystyle 2 0 {\displaystyle u >0} , where K ν {\displaystyle K_{u }} is the modified Bessel function of the second kind[1]Expected shortfall μ + s ((ν + [T − 1 (1 − p)] 2) × τ (T − 1 (1 − p)) (ν − 1) (1 − p)) , {\displaystyle \mu +s\left({\frac {{\big (}u +[T^{-1}(1-p)]^{2}{\big)}\times \tau {\big (}T^{-1}(1-p){\big)}}{(u -1)(1-p)}}\right),} where T − 1 {\displaystyle T^{-1}} is the inverse standardized Student t CDF, and τ {\displaystyle \tau } is the standardized Student t PDF.[2] In probability theory and statistics, Student's t distribution (or simply the t distribution) t ν {\displaystyle t_{u }} is a continuous probability distribution that generalizes the standard normal distribution. 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The Student's t distribution plays a role in a number of widely used statistical analyses, including Student's t-test for assessing the statistical significance of the difference between two sample means, the construction of confidence intervals for the difference between two population means, and in linear regression analysis. In the form of the location-scale t distribution ℓ s t (μ , τ 2 , ν) {\displaystyle \operatorname {\ell st} (\mu ,\tau ^{2},u)} it generalizes the normal distribution and also arises in the Bayesian analysis of data from a normal family as a compound distribution when marginalizing over the variance parameter. Student's t distribution has the probability density function (PDF) given by f (t) = Γ (ν + 1 2) π ν Γ (ν 2) (1 + t 2 ν) − (ν + 1) / 2 , {\displaystyle f(t)={\frac {\Gamma \left({\frac {u +1}{2}}\right)}{{\sqrt {\pi u }}\Gamma \left({\frac {u }{2}}\right)}}\left(1+{\frac {t^{2}}{u }}\right)^{-(u +1)/2},} where ν {\displaystyle u } is the number of degrees of freedom, and Γ {\displaystyle \Gamma } is the gamma function. This may also be written as f (t) = 1 ν B (1 2 , ν 2) (1 + t 2 ν) − (ν + 1) / 2 , {\displaystyle f(t)={\frac {1}{{\sqrt {u }}\,\mathrm {B} \left({\frac {1}{2}},{\frac {u }{2}}\right)}}\left(1+{\frac {t^{2}}{u }}\right)^{-(u +1)/2},} where B {\displaystyle \mathrm {B} } is the beta function. In particular for integer valued degrees of freedom ν {\displaystyle u } we have: For ν > 1 {\displaystyle u >1} and even, Γ (ν + 1 2) π ν Γ (ν 2) = 1 2 ν · (ν − 1) · (ν − 3) ⋯ 5 · 3 (ν − 2) · (ν − 4) ⋯ 4 · 2 . {\displaystyle {\frac {\Gamma \left({\frac {u +1}{2}}\right)}{{\sqrt {\pi u }}\,\Gamma \left({\frac {u }{2}}\right)}}={\frac {1}{2{\sqrt {u }}}}\cdot {\frac {(u -1)\cdot (u -3)\cdots 5\cdot 3}{(u -2)\cdot (u -4)\cdots 4\cdot 2}}.} For ν > 1 {\displaystyle u >1} and odd, Γ (ν + 1 2) π ν Γ (ν 2) = 1 π ν · (ν − 1) · (ν − 3) ⋯ 4 · 2 (ν − 2) · (ν − 4) ⋯ 5 · 3 . {\displaystyle {\frac {\Gamma \left({\frac {u +1}{2}}\right)}{{\sqrt {\pi u }}\,\Gamma \left({\frac {u }{2}}\right)}}={\frac {1}{\pi {\sqrt {u }}}}\cdot {\frac {(u -1)\cdot (u -3)\cdots 4\cdot 2}{(u -2)\cdot (u -4)\cdots 5\cdot 3}}.} The probability density function is symmetric, and its overall shape resembles the bell shape of a normally distributed variable with mean 0 and variance 1, except that it is a bit lower and wider. As the number of degrees of freedom grows, the t distribution approaches the normal distribution with mean 0 and variance 1. For this reason ν {\displaystyle {u }} is also known as the normality parameter.[3] The following images show the density of the t distribution for increasing values of ν . {\displaystyle u .} The normal distribution is shown as a blue line for comparison. Note that the t distribution (red line) becomes closer to the normal distribution as ν {\displaystyle u } increases. Density of the t distribution (red) for 1, 2, 3, 5, 10, and 30 degrees of freedom compared to the standard normal distribution (blue).Previous plots shown in green.1 degree of freedom2 degrees of freedom3 degrees of freedom5 degrees of freedom10 degrees of freedom30 degrees of freedom The cumulative distribution function (CDF) can be written in terms of I, the regularized incomplete beta function. For t > 0 , F (t) = ∫ − ∞ t f (u) d u = 1 − 1 2 I x (t) (ν 2 , 1 2) , {\displaystyle F(t)=\int _{-\infty }^{t}\ f(u)\ \operatorname {d} u~=~1-{\frac {1}{2}}I_{x(t)}\!\left({\frac {\ u \ }{2}},\ {\frac {\ 1\ }{2}}\right)\ ,} where x (t) = ν t 2 + ν . {\displaystyle x(t)={\frac {u }{\ t^{2}+u \ }}~.} Other values would be obtained by symmetry. An alternative formula, valid for t 2 < ν , {\displaystyle \ t^{2} 1 , {\displaystyle u >1\ ,} the raw moments of the t distribution are E { T k } = { 0 k odd , 0 < k < ν , 1 π Γ (ν 2) [Γ (k + 1 2) Γ (ν − k 2) ν k 2] k even , 0 < k < ν . {\displaystyle \operatorname {\mathbb {E} } \left\{\ T^{k}\ \right\}={\begin{cases}\quad 0&{\text{ odd }},\quad 04~.} Student's t-distribution with ν {\displaystyle u } degrees of freedom can be defined as the distribution of the random variable T with[5][6] T = Z V / ν = Z ν V , {\displaystyle T={\frac {Z}{\sqrt {V/u }}}=Z{\sqrt {\frac {u }{V}}},} where Z is a standard normal with expected value 0 and variance 1; V has a chi-squared distribution (χ2-distribution) with ν {\displaystyle u } degrees of freedom; Z and V are independent; A different distribution is defined as that of the random variable defined, for a given constant μ, by (Z + μ) ν V . {\displaystyle (Z+\mu){\sqrt {\frac {u }{V}}}.} This random variable has a noncentral t-distribution with noncentrality parameter μ. This distribution is important in studies of the power of Student's t-test. Suppose X1, ..., Xn are independent realizations of the normally-distributed, random variable X, which has an expected value μ and variance σ2. Let X ¯ n = 1 n (X 1 + ⋯ + X n) {\displaystyle {\overline {X}}_{n}={\frac {1}{n}}(X_{1}+\cdots +X_{n})} be the sample mean, and s 2 = 1 n − 1 ∑ i = 1 n (X i − X ¯ n) 2 {\displaystyle s^{2}={\frac {1}{n-1}}\sum _{i=1}^{n}\left(X_{i}-{\overline {X}}_{n}\right)^{2}} be an unbiased estimate of the variance from the sample. It can be shown that the random variable V = (n − 1) s 2 σ 2 {\displaystyle V=(n-1){\frac {s^{2}}{\sigma ^{2}}}} has a chi-squared distribution with ν = n − 1 {\displaystyle u =n-1} degrees of freedom (by Cochran's theorem).[7] It is readily shown that the quantity Z = (X ¯ n − μ) n σ {\displaystyle Z=\left({\overline {X}}_{n}-\mu \right){\frac {\sqrt {n}}{\sigma }}} is normally distributed with mean 0 and variance 1, since the sample mean X ¯ n {\displaystyle {\overline {X}}_{n}} is normally distributed with mean μ and variance σ2/n. Moreover, it is possible to show that these two random variables (the normally distributed one Z and the chi-squared-distributed one V) are independent. Consequently[clarification needed] the pivotal quantity T ≡ Z V / ν = (X ¯ n − μ) n s , {\textstyle T\equiv {\frac {Z}{\sqrt {V/u }}}=\left({\overline {X}}_{n}-\mu \right){\frac {\sqrt {n}}{s}},} which differs from Z in that the exact standard deviation σ is replaced by the sample standard error s, has a Student's t-distribution as defined above. Notice that the unknown population variance σ2 does not appear in T, since it was in both the numerator and the denominator, so it canceled. Gosset intuitively obtained the probability density function stated above, with ν {\displaystyle u } equal to n − 1, and Fisher proved it in 1925.[8] The distribution of the test statistic T depends on ν {\displaystyle u } , but not μ or σ; the lack of dependence on μ and σ is what makes the t-distribution important in both theory and practice. The t distribution arises as the sampling distribution of the t statistic. Below the one-sample t statistic is discussed, for the corresponding two-sample t statistic see Student's t-test. Let x 1 , … , x n ∼ N (μ , σ 2) {\displaystyle \ x_{1},\ldots ,x_{n}\sim {\mathcal {N}}(\mu ,\sigma ^{2})\ } be independent and identically distributed samples from a normal distribution with mean μ {\displaystyle \mu } and variance σ 2 . {\displaystyle \ \sigma ^{2}~.} The sample mean and unbiased sample variance are given by: x ¯ = x 1 + ⋯ + x n n , s 2 = 1 n − 1 ∑ i = 1 n (x i − x ¯) 2 . {\displaystyle {\begin{aligned}{\bar {x}}&={\frac {\ x_{1}+\cdots +x_{n}\ }{n}}\ ,\\[5pt]s^{2}&={\frac {1}{\ n-1\ }}\ \sum _{i=1}^{n}(x_{i}-{\bar {x}})^{2}~.\end{aligned}}} The resulting (one sample) t statistic is given by t = x ¯ − μ s / n ∼ t n − 1 . {\displaystyle t={\frac {{\bar {x}}-\mu }{\ s/{\sqrt {n\ }}\ }}\sim t_{n-1}~.} and is distributed according to a Student's t distribution with n − 1 {\displaystyle \ n-1\ } degrees of freedom. Thus for inference purposes the t statistic is a useful "pivotal quantity" in the case when the mean and variance (μ , σ 2) {\displaystyle (\mu ,\sigma ^{2})} are unknown population parameters, in the sense that the t statistic has then a probability distribution that depends on neither μ {\displaystyle \mu } nor σ 2 . {\displaystyle \ \sigma ^{2}~.} Instead of the unbiased estimate s 2 {\displaystyle \ s^{2}\ } we may also use the maximum likelihood estimate s M L 2 = 1 n ∑ i = 1 n (x i − x ¯) 2 {\displaystyle \ s_{\mathsf {ML}}^{2}={\frac {\ 1\ }{n}}\ \sum _{i=1}^{n}(x_{i}-{\bar {x}})^{2}\ } yielding the statistic t M L = x ¯ − μ s M L 2 / n = n n − 1 t . {\displaystyle \ t_{\mathsf {ML}}={\frac {{\bar {x}}-\mu }{\sqrt {s_{\mathsf {ML}}^{2}/n\ }}}={\sqrt {{\frac {n}{n-1}}\ }}\ t~.} This is distributed according to the location-scale t distribution: t M L ∼ ℓ s t (0 , τ 2 = n / (n − 1) , n − 1) . {\displaystyle t_{\mathsf {ML}}\sim \operatorname {\ell st} (0,\ \tau ^{2}=n/(n-1),\ n-1)~.} The location-scale t distribution results from compounding a Gaussian distribution (normal distribution) with mean μ {\displaystyle \ \mu \ } and unknown variance, with an inverse gamma distribution placed over the variance with parameters a = ν 2 {\displaystyle \ a={\frac {\ u \ }{2}}\ } and b = ν τ 2 2 . {\displaystyle b={\frac {\ u \ \tau ^{2}\ }{2}}~.} In other words, the random variable X is assumed to have a Gaussian distribution with an unknown variance distributed as inverse gamma, and then the variance is marginalized out (integrated out). Equivalently, this distribution results from compounding a Gaussian distribution with a scaled-inverse-chi-squared distribution with parameters ν {\displaystyle u } and τ 2 . {\displaystyle \ \tau ^{2}~.} The scaled-inverse-chi-squared distribution is exactly the same distribution as the inverse gamma distribution, but with a different parameterization, i.e. ν = 2 a , τ 2 = b a . {\displaystyle \ u =2\ a,\;{\tau }^{2}={\frac {\ b\ }{a}}~.} The reason for the usefulness of this characterization is that in Bayesian statistics the inverse gamma distribution is the conjugate prior distribution of the variance of a Gaussian distribution. As a result, the location-scale t distribution arises naturally in many Bayesian inference problems.[9] Student's t distribution is the maximum entropy probability distribution for a random variate X having a certain value of E { ln (ν + X 2) } {\displaystyle \ \operatorname {\mathbb {E} } \left\{\ \ln(u +X^{2})\ \right\}\ } .[10][clarification needed][better source needed] This follows immediately from the observation that the pdf can be written in exponential family form with ν + X 2 {\displaystyle u +X^{2}} as sufficient statistic. The function A(t | ν) is the integral of Student's probability density function, f(t) between -t and t, for t ≥ 0 . It thus gives the probability that a value of t less than that calculated from observed data would occur by chance. Therefore, the function A(t | ν) can be used when testing whether the difference between the means of two sets of data is statistically significant, by calculating the corresponding value of t and the probability of its occurrence if the two sets of data were drawn from the same population. This is used in a variety of situations, particularly in t tests. For the statistic t, with ν degrees of freedom, A(t | ν) is the probability that t would be less than the observed value if the two means were the same (provided that the smaller mean is subtracted from the larger, so that t ≥ 0). It can be easily calculated from the cumulative distribution function Fν(t) of the t distribution: A (t | ν) = F ν (t) − F ν (− t) = 1 − I ν ν + t 2 (ν 2 , 1 2) , {\displaystyle A(t\mid u)=F_{u }(t)-F_{u }(-t)=1-I_{\frac {u }{u +t^{2}}}\!\left({\frac {u }{2}},{\frac {1}{2}}\right),} where Ix(a, b) is the regularized incomplete beta function. For statistical hypothesis testing this function is used to construct the p-value. The noncentral t distribution generalizes the t distribution to include a noncentrality parameter. Unlike the nonstandardized t distributions, the noncentral distributions are not symmetric (the median is not the same as the mode). The discrete Student's t distribution is defined by its probability mass function at r being proportional to:[11] ∏ j = 1 k 1 (r + j + a) 2 + b 2 r = … , − 1 , 0 , 1 , … . {\displaystyle \prod _{j=1}^{k}{\frac {1}{(r+j+a)^{2}+b^{2}}}\quad \quad r=\ldots ,-1,0,1,\ldots ~.} Here a, b, and k are parameters. This distribution arises from the construction of a system of discrete distributions similar to that of the Pearson distributions for continuous distributions.[12] One can generate Student A(t | ν) samples by taking the ratio of variables from the normal distribution and the square-root of the χ² distribution. If we use instead of the normal distribution, e.g., the Irwin–Hall distribution, we obtain over-all a symmetric 4 parameter distribution, which includes the normal, the uniform, the triangular, the Student t and the Cauchy distribution. This is also more flexible than some other symmetric generalizations of the normal distribution. t distribution is an instance of ratio distributions. The square of a random variable distributed tn is distributed as Snedecor's F distribution F1,n. Student's t distribution generalizes to the three parameter location-scale t distribution ℓ s t (μ , τ 2 , ν) {\displaystyle \operatorname {\ell st} (\mu ,\ \tau ^{2},\ u)\ } by introducing a location parameter μ {\displaystyle \ \mu \ } and a scale parameter τ . {\displaystyle \ \tau ~.} With T ∼ t ν {\displaystyle \ T\sim t_{u }\ } and location-scale family transformation X = μ + τ T {\displaystyle \ X=\mu +\tau \ T\ } we get X ∼ ℓ s t (μ , τ 2 , ν) . {\displaystyle \ X\sim \operatorname {\ell st} (\mu ,\ \tau ^{2},\ u)~.} The resulting distribution is also called the non-standardized Student's t distribution. The location-scale t distribution has a density defined by:[13] p (x | ν , μ , τ) = Γ (ν + 1 2) Γ (ν 2) τ π ν (1 + 1 ν (x − μ τ) 2) − (ν + 1) / 2 {\displaystyle p(x\mid u ,\mu ,\tau)={\frac {\Gamma \left({\frac {u +1}{2}}\right)}{\Gamma \left({\frac {u }{2}}\right)\tau {\sqrt {\pi u }}}}\left(1+{\frac {1}{u }}\left({\frac {x-\mu }{\tau }}\right)^{2}\right)^{-(u +1)/2}} Equivalently, the density can be written in terms of τ 2 {\displaystyle \tau ^{2}} : p (x | ν , μ , τ 2) = Γ (ν + 1 2) Γ (ν 2) π ν τ 2 (1 + 1 ν (x − μ) 2 τ 2) − (ν + 1) / 2 {\displaystyle \ p(x\mid u ,\mu ,\tau ^{2})={\frac {\Gamma ({\frac {u +1}{2}})}{\Gamma ({\frac {u }{2}}){\sqrt {\pi u \tau ^{2}}}}}\left(1+{\frac {1}{u }}{\frac {(x-\mu)^{2}}{\tau ^{2}}}\right)^{-(u +1)/2}} Other properties of this version of the distribution are:[13] E { X } = μ for ν > 1 , var { X } = τ 2 ν ν − 2 for ν > 2 , mode { X } = μ . {\displaystyle {\begin{aligned}\operatorname {\mathbb {E} } \{\ X\ \}&=\mu &{\text{ for }}u >1\ ,\\\operatorname {var} \{\ X\ \}&=\tau ^{2}{\frac {u }{u -2}}&{\text{ for }}u >2\ ,\\\operatorname {mode} \{\ X\ \}&=\mu ~.\end{aligned}}} If X {\displaystyle \ X\ } follows a location-scale t distribution X ∼ ℓ s t (μ , τ 2 , ν) {\displaystyle \ X\sim \operatorname {\ell st} \left(\mu ,\ \tau ^{2},\ u \right)\ } then for ν → ∞ {\displaystyle \ u \rightarrow \infty \ } X {\displaystyle \ X\ } is normally distributed X ∼ N (μ , τ 2) {\displaystyle X\sim \mathrm {N} \left(\mu ,\tau ^{2}\right)} with mean μ {\displaystyle \mu } and variance τ 2 . {\displaystyle \ \tau ^{2}~.} The location-scale t distribution ℓ s t (μ , τ 2 , ν = 1) {\displaystyle \ \operatorname {\ell st} \left(\mu ,\ \tau ^{2},\ u =1\right)\ } with degree of freedom ν = 1 {\displaystyle u =1} is equivalent to the Cauchy distribution C a u (μ , τ) . {\displaystyle \mathrm {Cau} \left(\mu ,\tau \right)~.} The location-scale t distribution ℓ s t (μ = 0 , τ 2 = 1 , ν) {\displaystyle \operatorname {\ell st} \left(\mu =0,\ \tau ^{2}=1,\ u \right)\ } with μ = 0 {\displaystyle \mu =0} and τ 2 = 1 {\displaystyle \ \tau ^{2}=1\ } reduces to the Student's t distribution t ν . {\displaystyle \ t_{u }~.} Student's t distribution arises in a variety of statistical estimation problems where the goal is to estimate an unknown parameter, such as a mean value, in a setting where the data are observed with additive errors. If (as in nearly all practical statistical work) the population standard deviation of these errors is unknown and has to be estimated from the data, the t distribution is often used to account for the extra uncertainty that results from this estimation. In most such problems, if the standard deviation of the errors were known, a normal distribution would be used instead of the t distribution. Confidence intervals and hypothesis tests are two statistical procedures in which the quantiles of the sampling distribution of a particular statistic (e.g. the standard score) are required. In any situation where this statistic is a linear function of the data, divided by the usual estimate of the standard deviation, the resulting quantity can be rescaled and centered to follow Student's t distribution. Statistical analyses involving means, weighted means, and regression coefficients all lead to statistics having this form. Quite often, textbook problems will treat the population standard deviation as if it were known and thereby avoid the need to use the Student's t distribution. These problems are generally of two kinds: (1) those in which the sample size is so large that one may treat a data-based estimate of the variance as if it were certain, and (2) those that illustrate mathematical reasoning, in which the problem of estimating the standard deviation is temporarily ignored because that is not the point that the author or instructor is then explaining. A number of statistics can be shown to have t distributions for samples of moderate size under null hypotheses that are of interest, so that the t distribution forms the basis for significance tests. For example, the distribution of Spearman's rank correlation coefficient ρ, in the null case (zero correlation) is well approximated by the t distribution for sample sizes above about 20.[citation needed] Suppose the number A is so chosen that P { − A < T < A } = 0.9 , {\displaystyle \ \operatorname {\mathbb {P} } \left\{\ {-A}

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