I'm not a robot

Written By Uma A V Last Modified 22-06-2023 Increasing and Decreasing Functions: Any activity can be represented using functions, like the path of a ball followed when thrown. If you have the position of the ball at various intervals, it is possible to find the rate at which the position of the ball is changing. If we graph the path the ball follows with respect to time, we can see the change in the position at various time intervals. Such studies of derivatives have practical applications in multiple fields. One of the most important applications is to study the monotonicity of the function on its domain. In this article, we will learn about increasing and decreasing functions in detail. The relation \(R\) from the non-empty set \(A\) to the non-empty set \(B\) is a subset of the cartesian product \(A×B.\) It is represented as shown below. Here, the relation \(R\) is the set of ordered pairs that is written as: \(\{ (1,5),(3,15),(5,25),(7,35)\} \) A relation f from a set \(A\) to a set \(B\) is a function if every element of set \(A\) has one and only one image in set \(B.\) If you consider the relation \(R\) as shown above, element \(10\) in set \(A\) does not have an image in set \(B,\) and therefore, \(R\) is not a function. If we map the element \(10\) in set \(A\) to \(50\) in set \(A\) to \(50\) in set \(B,\) the relation \(R^\) given by: \(\{ (1,5),(3,15),(5,25),(7,35),(10,50)\} \) \(R^\) is a function defined by the equation \(y=5x,\) where \(x\) is the independent variable whose values vary within the elements of the set \(A\) and \(y\) is the dependent variable whose values range within the elements of the set \(B.\) The derivative of a real-valued function measures the tendency of a function to change the values with respect to the change in its independent variable. The derivative of a function at a point, if exists, is the slope of the tangent line to the graph of the function at that point. If \(f(x)\) is a real-valued function that is differentiable at a point a, and if the domain contains an open interval \(I\) containing \(a\) and the limit \(\frac{ f(a + h) − f(a)}{h}\) exists, this limit is called the derivative of the function \(f(x)\) at \(a.\) There are many applications of the derivative of a function. One of the most important uses is to understand the monotonicity of the function. That is, it can help you know whether a function is increasing or decreasing or stays constant at a point or in an interval. As the word suggests, a function is said to be increasing when the value of the dependent variable \(y\) increases with \(x.\) Example 1: Consider the graph of the function \(y=5x.\) Observe that, as the value of \(x\) increases, the corresponding \(y\) values also increase. So, \(y\) is an increasing function. Example 2: Consider the function \(y = {e^x}\) as an increasing function as the \(y\) values increase with increasing \(x\) values. Contrary to the increasing functions, a function is said to be decreasing when the values of the dependent variable \(y\) decrease as \(x\) increases. Example 1: Consider the function \(y=-2x.\) \(x\)\(-2\)\(-1\)\(0\)\(1\)\(3\)\(y=f(x) =-2x\)\(4\)\(2\)\(0\)\(-2\)\(-4\)\(-6\)\) Observe that, as the value of \(x\) increases from \(-2\) to \(3,\) the corresponding \(y\) value decreases from \(4\) to \(-6.\) So, \(y\) is a decreasing function. Example 2: The function \(y =\, - \log x\) is a decreasing function as the \(y\) values decrease with increasing \(x\) values. Some functions may be increasing or decreasing at particular intervals. Example: Consider a quadratic function \(y = {x^2}.\) Observe that the dependent variable y decreases as the independent variable \(x\) increases in the interval \((-∞,0)\) whereas \(y\) increases with \(x\) in the interval \((0, ∞).\) This function is decreasing in the interval \((-∞,0)\) and increasing in the interval \((0, ∞).\) Let us now define increasing and decreasing functions systematically. Let \(f\) be an interval contained in the domain of a real-valued function \(f.\) Then \(f\) is said to be: (i) increasing on \(f\) if \({x_1} < {x_2}\) in \(I.\) This implies \(f\left({x_1} \right) \le f\left({x_2} \right)\) for all \({x_1}, {x_2} \in I.\) (ii) strictly increasing on \(f\) if \({x_1} < {x_2}\) in \(I.\) This implies \(f\left({x_1} \right) < f\left({x_2} \right)\) for all \({x_1}, {x_2} \in I.\) Let us now define strictly decreasing functions systematically. Let \(f\) be an interval contained in the domain of a real-valued function \(f.\) Then \(f\) is said to be: (i) decreasing on \(f\) if \({x_1} < {x_2}\) in \(I.\) This implies \(f\left({x_1} \right) \ge f\left({x_2} \right)\) for all \({x_1}, {x_2} \in I.\) (ii) strictly decreasing on \(f\) if \({x_1} < {x_2}\) in \(I.\) This implies \(f\left({x_1} \right) > f\left({x_2} \right)\) for all \({x_1}, {x_2} \in I.\) A function \(f\) is said to be increasing at a particular point \({x_0},\) if there exists an open interval \(I\) containing \({x_0}\) such that \(f\) is increasing in \(I.\) A function \(f\) is said to be decreasing at a particular point \({x_0},\) if there exists an open interval \(I\) containing \({x_0}\) such that \(f\) is decreasing in \(I.\) Example: The function \(y = {x^2}\) is decreasing at \(x=-2\) and increasing at \(x=2\). How to identify the nature of a function at a given point? Let us look into a theorem related to it. Let \(f\) be continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a,b).\) Then: Case 1: \(f\) is increasing in \([a,b],\) if \(f'(x)>0,\) for each \(x\in(a,b).\) Case 2: \(f\) is decreasing in \([a,b],\) if \(f'(x)<0\)Therefore, by the increasing function theorem, the function is increasing.If you look at the graph of the function, it is clear that it is an increasing function. Q.3. Find the least value of a for which the function \(g(x) = {x^2} + ax + 5\) is increasing on the interval \([0, 1].\)Ans: The first derivative of the function is:\(g(x)=2x+a\)For the extreme values of the interval \([0, 1],\) the first derivatives are:\(a=0:\)\(g'(x)=2x+0\)\(g'(x)=2x\)\(a=1:\)\(g'(x)=2x+1\)For the function to be increasing, these values should be greater than zero.That is,\(g'(x)=2x=2>0\)\(g'(x)=2x + 1 > 0 \Rightarrow x > - \frac{1}{2}\)Therefore, the least value of a for which the function \(g(x)\) increases is theinterval \([0, 1]\) is \(0.\) Q.4. Find the interval in which the function f increases or decreases where f is given by \(f(x) = \cos 2x\) for \(- \frac{\pi }{4} \le x \le \frac{\pi }{4}\)Ans: \(f'(x) = - \sin (2x) \times 2\) for \(- \frac{\pi }{4},\frac{\pi }{4} \right]\)So, the point \(x=0\) divides the interval \(x \in \left[{ - \frac{\pi }{4},\frac{\pi }{4} } \right]\) into two disjoint intervals \(\left[{ - \frac{\pi }{4},0} \right]\) and \(\left({0,\frac{\pi }{4} } \right]\)Now, \(f'(x)<0\)\(- \sin 2x < 0\)\(\sin 2x = 0 \to \sin 2x = 0 \to 0 \to x \in \left[{ - \frac{\pi }{4},\frac{\pi }{4} } \right]\)Therefore, the function is increasing for all the values in the domain. The article helps you understand the basic concepts of functions right from their definitions. Then the article defines the derivative of a function and applies the concept of derivatives in finding the monotonicity of a function. The concepts of natures of increasing, strictly increasing, decreasing, strictly decreasing, and constant functions are explained with examples. A theorem as an application of derivatives is also discussed in simple language for better understanding. Further, the characteristics of functions such as local maxima, local minima, critical points are well explained with the first and second derivative tests to find these points in a real-valued function. Finally, a few solved examples are worked out to clarify the concepts explained. Q.1. What is meant by increasing and decreasing function?Ans: A function is "increasing" when the dependent variable \(y\) increase as that of the independent variable \(x\) increase. A function is "decreasing" when the values of the dependent variable \(y\) decrease as that of the independent variable \(x\) increase. Q.2. What is the difference between increasing and strictly increasing function?Ans: Let \(f\) be an interval contained in the domain of a real-valued function \(f.\)Then \(f\) is said to be:increasing on \(f\) if \({x_1}, {x_2} \in I.\) This implies \(f\left({x_1} \right) \le f\left({x_2} \right)\) for all \({x_1}, {x_2} \in I.\)Strictly increasing on \(f\) if \({x_1}, {x_2} \in I.\) This implies \(f\left({x_1} \right) < f\left({x_2} \right)\) for all \({x_1}, {x_2} \in I.\)Q.3. What is strictly decreasing function?Ans: Let \(f\) be an interval contained in the domain of a real-valued function \(f.\)Then \(f\) is said to be strictly decreasing on \(f\) if \({x_1} < {x_2}\) in \(I.\) This implies \(f\left({x_1} \right) > f\left({x_2} \right)\) for all \({x_1}, {x_2} \in I.\)Q.4. Which functions are always increasing?1. Linear2. Quadratic3. Absolute Value4. Square Root5. Cubic6. Cube Root7. Rational8. ExponentialAns: Let \(f\) be an interval contained in the domain of a real-valued function \(f.\)Then \(f\) is said to be increasing on \(f\) if \({x_1} < {x_2}\) in \(I.\) This implies \(f\left({x_1} \right) < f\left({x_2} \right)\) for all \({x_1}, {x_2} \in I.\)Let \(f\) be continuous on the closed interval \([a, b]\) and differentiable on the open interval \((a,b).\) Then, \(f\) is increasing in \([a, b],\) if \(f'(x)>0\) for each \(x\in(a,b).\) Out of the given functions, only the cubic functions always have the positive first derivatives, thus making the functions always increasing. Q.5. How do you find the increasing and decreasing intervals?Ans: First, identify the value of the independent variable in the interval for which the first derivative is zero. Now, the function is increasing on the interval where the first derivative is positive, and it is decreasing where the first derivative is negative. We hope you find this article on 'Increasing and Decreasing Functions' helpful. In case of any queries, you can reach back to us in the comments section, and we will try to solve them. Increasing and decreasing functions refer to the behavior of a function's graph as you move from left to right along the x-axis. A function is considered increasing if for any two values x1 and x2 such that x1 < x2, the function value at x1 is less than the function value at x2 (i.e., f(x1) < f(x2)). On the other hand, a function is decreasing if f(x1) > f(x2) for x1 < x2.Some real-life examples of increasing/decreasing functions are below:Increasing Function Definition:In simple words, an increasing function is a type of function where, with increasing input (or the independent variable), output also increases (or the value of the function). Now, let's define an increasing function formally.Now, let us consider f to be an interval that is present in the domain of a real-valued function f, then the function f is increasing on I.if x1 < x2 = f(x1) ≤ f(x2) ∀ x1 and x2 ∈ ISome common examples of increasing functions include linear functions with positive slope (such as y = mx + b), exponential functions (such as y = ax, where a is a positive constant), and power functions (such as y = xn, where n is a positive integer).Solved Example: Consider the function: f(x) = 2x + 3. If x1 = 2 and x2 = 5.Determine whether the function is increasing or decreasing.Solution:A function is considered increasing if, for any two values x1 and x2 such that we have f(x1) < f(x2).Given that x1 = 2, x2 = 5.f(x1) = f(2) = 2(2) + 3 = 4 + 3 = 7f(x2) = f(5) = 2(5) + 3 = 10 + 3 = 13Since, the function satisfies the definition of an increasing function.Therefore, f(x) = 2x + 3 is increasing.Strictly Increasing FunctionFor a function to be strictly increasing, the function should be increasing, but it can't be equal for any two unequal values, i.e.,if x1 < x2 = f(x1) < f(x2) ∀ x1 and x2 ∈ IDecreasing Function Definition:In simple words, a decreasing function is a type of function where, with increasing input (or the independent variable), the output value decreases (or the value of the function). To define a decreasing function formally, let us consider I to be an interval that is present in the domain of a real-valued function f, then the function f is decreasing on I.if x1 < x2 = f(x1) ≥ f(x2) ∀ x1 and x2 ∈ ISome common examples of decreasing functions include exponential decay functions (such as y = a^(-x), where a is a positive constant), and negative power functions (such as y = x^(-n), where n is a positive integer).Solved Example: Determine whether the function f(x) = -2x + 5 is decreasing, if x1 = 1 and x2 = 3.Solution:A function is considered decreasing if f(x1) > f(x2) for x1 < x2.Since x1 < x2, we now calculate f(x1) and f(x2):f(x1) = f(1) = −2(1) + 5 = −2 (1) + 5 = 3f(x2) = f(3) = −2(3) + 5 = −6 + 5 = −1Clearly, f(1) > f(3) So as x increases, f(x) decreases. This satisfies the condition for a strictly decreasing function.Strictly Decreasing FunctionFor a function to be strictly decreasing, the function should be decreasing, but it can't be equal for any two unequal values, i.e.,if x1 < x2 = f(x1) > f(x2) ∀ x1 and x2 ∈ IConstant FunctionIn simple words, a constant function has more formally, the output always remains the same, i.e., for all the inputs output remains constant. To define a constant function more formally, a function f is said to be a constant function if and only iff(x) = kWhere k is the real number. ➤ Read all About Increasing and Decreasing in Maths - [Read Here!]Rules to Check Increasing and Decreasing FunctionsIn calculus, an increasing function can be defined in terms of the slope of any curve, as an increasing function always has a positive slope, i.e., dy/dx > 0. To define an increasing function more formally, let us consider f to be a function that is continuous on the interval [p, q] and differentiable on the open interval (p, q), thenFunction f is increasing in [p, q] if f'(x) > 0 for each x ∈ (p, q).As a decreasing function always has a negative slope, a decreasing function can be defined in terms of the slope of any curve, i.e., dy/dx < 0. For a more formal definition of the decreasing function, let us consider f to be a function that is continuous on the interval [p, q] and differentiable on the open interval (p, q), thenFunction f is decreasing in [p, q] if f'(x) < 0 for each x ∈ (p, q).Graph of Increasing, Decreasing, and Constant FunctionsThe graphical representation of an increasing function, a decreasing function, and a constant function is,Example: In this example, we will investigate the graph of f(x) = x2. Solution:Function table: x-4-3-2-101234f(x)16941014916As we can see that when x < 0, the value of f(x) is decreasing as the graph moves to the right. In other words, the "height" of the graph is getting smaller. This is also confirmed by looking at the table of values. When x < 0, as x increases, f(x) decreases. Therefore, f(x) is decreasing on the interval from negative infinity to 0. When x > 0, the opposite is happening. When x > 0, the value of f(x) is increasing as the graph moves to the right. In other words, the "height" of the graph is getting bigger. This is also confirmed by looking at the table of values. When x > 0, as x increases, f(x) increases. Therefore, f(x) is increasing on the interval from 0 to infinity. Properties of Increasing & Decreasing FunctionsSome helpful algebraic properties of increasing & Decreasing Functions are as follows:Additive property: If the functions f and g are increasing/decreasing on the interval (a, b), then the sum of the functions f + g is also increasing/decreasing on this interval.Opposite property: If the function f is increasing/decreasing on the interval (a, b), then the opposite function, -f, is decreasing/increasing.Inverse function, If f, is decreasing/increasing on this interval.Multiplicative property: If the functions f and g are increasing/decreasing and are not negative on the interval (a, b), then the product of the functions is also increasing/decreasing.How to Find Increasing and Decreasing Intervals?Given a function, f(x), we can determine the intervals where it is increasing and decreasing by using differentiation and algebra. Step 1: Find the derivative, f'(x), of the function. Step 2: Find the zeros of f'(x). Remember, zeros are the values of x for which f'(x) = 0. Set f'(x) = 0 and solve for x. Step 3: Determine the intervals. The intervals are between the endpoints of the interval of f(x) and the zeros of f'(x). If the interval of f(x) is not given, assume f(x) is on the interval (-∞, ∞). Step 4: Determine whether the function is increasing or decreasing on each interval. Given the interval (a, c), choose a value b, a < b < c. Solve for f'(b). If f'(b) is positive, f(x) is increasing on (a, c).Example 1: If g(x) = (x - 5)2, find the intervals where g(x) is increasing and decreasing. Solution:Step 1: Find the derivative of the function. Using the chain rule, g'(x) = 2(5 - x)Step 2: Find the zeros of the derivative function. In other words, find the values for which g(x) equals zero. You can do this by setting g(x) = 0 and using algebra to solve for x. From the definitions above, we know the function is constant at points where the derivative is zero. g'(x) = 0 = 2(5 - x) = 0 = 5 - x = x = 5Step 3: Use the zeros to determine intervals. Since x = 5 is the only zero for g'(x), there are just 2 intervals: from negative infinity to 5, and from 5 to negative infinity. These can be denoted in inequality notation: -∞ < x < 5= 5 < x < ∞Or in interval notation: (-∞, 5), (5, ∞)Remember, the endpoints are NOT inclusive because g(x) is neither increasing nor decreasing at these points. Step 4: Determine if the function is increasing or decreasing in each interval.For the first interval, ((-∞, 5), we'll choose b = 0. -∞ < x < 5g'(b) = g'(0) = 2(5-0) = 10 10 > 0 POSITIVEFor the second interval, (5, ∞), we'll choose b = 6. 5 < 6 < ∞g'(b) = g'(6) = 2(5-6) = -2 -2 < 0 NEGATIVETherefore, g(x) is increasing on (-∞, 5) and decreasing on (5, ∞). We can verify our results visually. In the graph below, you can clearly see that f(x) = (x - 5)2 is increasing on the interval (5, ∞) and decreasing on the interval (-∞, 5).We can visually verify our result by investigating the graph of g(x). Looking at the graph, g(x) is indeed increasing in the interval from negative infinity to 5 and decreasing in the interval from 5 to infinity. Example 2: Find the intervals on -20 < x < 20 where g(x) is increasing and decreasing, given g'(x) = x2 - 100.Solution:We know that for a differentiable function f(x) is increasing or decreasing. Solution: We know that f(x) > 0 and we need to have f'(x) > 0 for all x in I and or a differentiable function f(x) to be decreasing on an interval I, we need to have f'(x) < 0 for all x in I. As we can see in the image above, the graph of f(x) = 0 (above x-axis) in the interval (-2, 2) and the graph of f(x) < 0 (below x-axis) in the intervals (-∞, -2) and (2, ∞). Therefore, the function f(x) is increasing in (-2, 2) and decreasing in (-∞, -2) and (2, ∞). View Answer > go to slidego to slide Great learning in high school using simple cues Indulging in rote learning, you are likely to forget concepts. With Cuemath, you will learn visually and be surprised by the outcomes. Book a Free Trial Class FAQs on Increasing and Decreasing Functions Increasing and decreasing functions are defined as: A function f(x) is said to be increasing on an interval I, then we know f(x) > f(y) for any two numbers x and y in I such that x < y, we have f(x) ≥ f(y). Decreasing Function A function f(x) is said to be decreasing on an interval I. Which Function is Neither Increasing Nor Decreasing? A constant function is neither increasing nor decreasing as the graph of a constant function is a straight line parallel to the x-axis and its derivative is always 0. How to Find where Functions are Increasing and Decreasing Functions? Functions can be identified as increasing and decreasing functions using differentiation. we can find the derivative of the function and determine its critical point. Around the critical point, we can check the sign of the derivative and determine if it is increasing or decreasing. How to Find Intervals of Increasing and Decreasing Functions? Intervals of increasing and decreasing functions can be calculated using differentiation. we can find the derivative of the function and determine its critical point. Around the critical point, we can check the sign of the derivative and determine if it is increasing or decreasing. Suppose a function f(x) is differentiable on an interval I. If f'(x) ≥ 0 on I, the function is said to be an increasing function on I. If f'(x) ≤ 0 on I, the function is said to be a decreasing function on I. How to Identify Increasing and Decreasing Functions Through Graph? As move towards, the right-hand side of the x-axis (increase in value of x), if the graph of f(x) goes upwards or remains constant, then f(x) is increasing and if the graph of f(x) goes downwards or remains constant, then f(x) is decreasing and if the graph of f(x) goes downwards or remains constant, then f(x) is decreasing. Intervals of increasing and decreasing functions are intervals of real numbers where the real-valued functions are increasing and decreasing respectively. To determine the increasing and decreasing intervals, we use the first-order derivative test to check the sign of the derivative in each interval. Therefore it is increasing if the value of the function f(x) increases with an increase in the value of x and it is decreasing if f(x) decreases with a decrease in x. In this article, we will learn to determine the increasing and decreasing intervals using the first-order derivative test and the graph of the function with the help of examples for a better understanding of the concept. What are Increasing and Decreasing Intervals? The intervals where the functions are increasing or decreasing are called the increasing and decreasing intervals. These intervals can be evaluated by checking the sign of the first derivative of the function in each interval. If the first derivative of a function is positive in an interval, then it is said to be an increasing interval and if the first derivative of the function is negative in an interval, then it is said to be a decreasing interval. Finding Increasing and Decreasing Intervals Now, we have understood the meaning of increasing and decreasing intervals, let us now learn how to calculate increasing and decreasing intervals. We will use an example to understand the concept. Consider f(x) = x3 + 3x2 - 45x + 9. Differentiating/Increasing with respect to x to find f'(x): f'(x) = 3x2 + 6x - 45 = 3(x2 + 2x - 15) = 3(x + 5)(x - 3) Substitute f'(x) = 0 = x = -5, x = 3 Now, the x-intercepts of f'(x) are x = -5 and x = 3. The intervals that we have are (-∞, -5), (-5, 3), and (3, ∞). We will check the sign of f'(x) in each of these intervals to identify increasing and decreasing intervals. Interval Value of x f'(x) Increasing/Decreasing (-∞, -5) x = -6 f'(-6) = 27 > 0 Increasing (-5, 3) x = 0 f'(0) = -45 < 0 Decreasing (3, ∞) x = 4 f'(4) = 27 > 0 Increasing Hence, the increasing intervals for f(x) = x3 + 3x2 - 45x + 9 are (-∞, -5) and (3, ∞), and the decreasing interval of f(x) is (-5, 3). Increasing and decreasing intervals are intervals of real numbers where the real-valued functions are increasing and decreasing respectively. The first graph shows an increasing function as the graph goes upwards as we move from left to right along the x-axis. The second graph shows a decreasing function as the graph moves downwards as we move from left to right along the x-axis. Important Notes on Increasing and Decreasing Intervals For a real-valued function f(x), the interval I is said to be a strictly increasing interval if for every x < y, we have f(x) < f(y). For a real-valued function f(x), the interval I is said to be a strictly decreasing interval if for every x < y, we have f(x) > f(y). The function is constant in an interval if f'(x) = 0 through that interval. Related Topics Applications of Derivatives Differential Equations Calculus Example 1: Determine the increasing and decreasing intervals for the function f(x) = -x3 + 3x2 + 9. Solution: Differentiate f(x) = -x3 + 3x2 + 9 w.r.t. x. f'(x) = -3x2 + 6x = -3x(x - 2) = f'(x) = 0 = -3x(x - 2) = 0 = x = 0, or x = 2 The intervals that we have are (-∞, 0), (0, 2), and (2, ∞). We need to identify the increasing and decreasing intervals from these. Interval Value of x f'(x) Increasing/Decreasing (-∞, 0) x = -1 f'(-1) = -9 < 0 Decreasing (0, 2) x = 1 f'(1) = 3 > 0 Increasing (2, ∞) x = 4 f'(4) = -24 < 0 Decreasing Answer: Hence, (-∞, 0) and (2, ∞) are intervals of increase and decrease respectively. Example 2: Show that (-∞, ∞) is a strictly increasing interval for f(x) = 3x + 5. Solution: Consider two real numbers x and y in (-∞, ∞) such that x < y. Answer: Hence, (-∞, ∞) is a strictly increasing interval for f(x) = 3x + 5. View Answer > go to slidego to slide Breakdown tough concepts through simple visuals. Math will no longer be a tough subject, especially when you understand the concepts through visualizations. Book a Free Trial Class FAQs on Increasing and Decreasing Intervals Increasing and decreasing intervals are intervals of real numbers where the real-valued functions are increasing and decreasing respectively. Why are Only the X-values Used in Describing Increasing and Decreasing Intervals of a Function? We can find increasing and decreasing intervals of a function using its first derivative. We can find the critical points and hence, the intervals. Then, we can check the sign of the derivative in each interval to identify increasing and decreasing intervals. How to Find Increasing and Decreasing Intervals Using Graph? How do you Know When a Function is Increasing? A function f(x) is said to be increasing on an interval I if for any two numbers x and y in I such that x < y, we have f(x) ≤ f(y). Which Function Does Not Have Increasing and Decreasing Intervals? A constant function is neither increasing nor decreasing.) To find an interval in which a function has a decreasing function, we need to find the range of values of x for which f(x) . To find an interval of decrease for . First, differentiate. Then put less than zero and solve: So we know that the function is decreasing when. Let us now put the original function into Desmos (opens in new tab). For each of the functions listed below, find the first derivative and, hence, find the interval of increase. Click here to return to the main menu. You might sometimes be asked to explain why a function is always increasing or decreasing). To do this, you need to show that will always be positive (or negative). The most likely scenario where you'll encounter this type of question is with a cubic function. If the original function is a cubic then of course will be a quadratic; you then have a choice of approaches: You can put into completed square form and read off the minimum value of the function (or the maximum if it's a "sad face" curve), or You can use the discriminant to show that the gradient curve is always above (or below) the x-axis. Example: Demonstrate that the function is increasing for all real values of x. First, find the first derivative: . In completed square form this is , which tells us that the minimum value of is 3 (and that this occurs when x = 2). If can never be less than 3 then the gradient is always positive for all values of x. Alternatively, using the discriminant, b2 - 4ac = (4)2 - 4(1)(7) = 16 - 28 = -12.Since b2 - 4ac < 0, the curve does not cross the x-axis at any point. Since the x2 term in is positive, this must be above the x-axis, so that the whole gradient curve is above the x-axis, and so the minimum gradient must be positive. Therefore the original function is increasing for all values of x. You can also use the discriminant method to find a range of values for an unknown coefficient that will make the function increasing for all real values of x. Example: Given that the function is increasing for all real values of x , find the range of possible values for . Solution: For an always-increasing gradient, we need b2 - 4ac > 0 so that the entire gradient curve is above the x-axis).b2 - 4ac = (2k)2 - 4(1)(9) = 4k2 - 36, < 0 = k2 < 9 = -3 < k < 3 = -3 = k = ± 3 3Know whether a function is increasing or decreasing, we have to know where the graph of a function rises and where it falls. The graph shown below rises, falls, then rises again as we move from left to right: It rises from A to B, falls from B to C, and rises again from C to D. The function f is said to be increasing when its graph rises and decreasing when its graph falls In the above graph, f is increasing on [a, b] and [c, d]f is decreasing on [b, c] Increasing Function :A function f is called increasing on an interval I, iff(x1) ≤ f(x2)whenever x1