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Circle theorems summary

Theorems This segment delves into circular theorem concepts, covering tangents, sectors, angles, and evidence. The accompanying video outlines essential rules for calculating circular theorems. Isosceles Triangle An isosceles triangle forms when two radii and a chord intersect. Perpendicular Chord Bisection A perpendicular line drawn from the center of a circle to a chord will consistently bisect it (dividing it into two equal lengths). Angles Subtended on the Same Arc Angles created by two points on the circumference are equivalent to other angles, within the same arc, formed by those two points. Angle in a Semi-Circle Angles produced by drawing lines from a diameter's ends to its circumference form a right angle (90 degrees), making c a right angle. Proof The original triangle can be divided into two equal parts by drawing a line from the circle's center to its point of contact. Given that each radius is identical, the resulting triangles are isosceles and have corresponding pairs of equal angles. As these angles collectively total 180° (the sum of a triangle's internal angles), it follows that x + y + x + y = 180 (2(x + y) = 180). Therefore, x + y equals 90 degrees. Tangents A tangent to a circle is a straight line that touches the circle at only one point (without crossing it). At this point of contact, a tangent forms a right angle with the circle's radius. Additionally, when two tangents intersect on a circle, their lengths are equal from the point of contact to where they cross. Angle at the Centre The angle formed by lines originating from two points on the circumference and meeting at the center is double the angle created on the circumference by lines from the same points (a = 2b). Proof Since both OA and OX represent a circle's radius, we can equate them as equal. This makes triangle AOX an isosceles triangle with angles ∠OXA = a and ∠OXB = b. Given that a triangle's internal angles sum to 180°, ∠XOA equals 180 - 2a. Similarly, ∠BOX = 180 - 2b. Knowing the total angle around a point is 360°, we find ∠AOB by combining these: ∠AOB = 360 - (180 - 2a) - (180 - 2b). This simplifies to 2(a + b), or 2 times the angle formed on the circumference. Alternate Segment Theorem In this diagram, we see that red and green angles are equal pairs. Proof Using related angle properties: a tangent makes a 90-degree angle with the radius of a circle (so ∠OAC + x = 90). Since an angle in a semi-circle is also 90 degrees (∠BCA = 90), and the sum of triangle angles is 180, ∠BCA + ∠OAC + y = 180. Simplifying gives us 90 + ∠OAC + y = 180 and ultimately ∠OAC + x = 90. Given that ∠OAC + x = 90 and ∠OAC + y = 90, it follows that x equals y (hence, x = y). Cyclic Quadrilaterals A cyclic quadrilateral is a four-sided figure where each vertex touches the circle's circumference. The circle is a fundamental concept in geometry, where opposite angles add up to 180 degrees. The area of a sector and arc length can be calculated using specific formulas. Given article text here The angle subtended by a chord can be defined as the angle formed at a point on the circle by two given points on the circle. The central angle is an angle with its vertex at the center of a circle and whose sides are radii intersecting the circle in two distinct points. Central angles, inscribed angles, and chords are closely related geometric concepts. The first theorem about circles, known as Thales' Theorem, was developed around 650 BC. Engineers and designers use these circle theorems to calculate angles, arcs, and their relationships within a circle. Understanding these theorems is crucial for various geometric applications. In this article, we discussed the essential circle theorems that provide valuable insights into geometry. These theorems are based on properties of circles such as tangents, chords, and central angles. For instance, the alternate segment theorem states that the angle formed between a tangent and a chord is equal to the angle in the alternate segment. The circle theorems also state that the sum of interior angles of a triangle is 180 degrees. The figure given below shows an inscribed triangle ABC in a circle with a tangent DE meeting the circle at point B. We can use the circle theorems to find the measure of angle x and y. Since the sum of interior angles of a triangle is 180, we know that x + 57 + 48 = 180, which gives us x = 75. According to the alternate segment theorem, the angle formed between the tangent and the chord through point B is equal to the angle in the alternate segment. Therefore, x = y = 75. The figure given below shows an angle ABC in a semicircle. According to the circle theorems, the angle in a semicircle is always a right angle. So, m∠ABC = 90. We also know that the sum of interior angles of a triangle is 180, which gives us x + 37 + 90 = 180. Solving for x, we get x = 53. The figure given below shows an angle AOB subtended by two points on a circle. According to the angle at the center theorem, the central angle subtended by two points on a circle is always twice the inscribed angle subtended by those points. Therefore, ∠AOB = 2∠ACB, which gives us x = 470. The figure given below shows a point O as the center of a circle with radius 17 inches and OP = 8 inches. We can use the Pythagorean theorem to find the length of chord AB. Since OP is perpendicular to AB, we have OP^2 + PB^2 = OB^2. Solving for PB, we get PB = √225 = 15 inches. According to the chord of a circle theorem, the perpendicular drawn from the center of the circle bisects the chord. Therefore, AP = PB, which gives us AB = AP + PB = 30 inches. Hence, the length of the chord AB is 30 inches. Given text: paraphrase this text The value of “x” and “y” can be determined using the cyclic quadrilateral theorem. In a cyclic quadrilateral, the sum of opposite angles is equal to 180 degrees. So, x + 980 = 1800, which gives x = 820. Similarly, y + 950 = 1800, resulting in y = 850. The same principle applies to finding the value of “x” in a given figure. Two tangent lines PA and PB are drawn from an external point P. Since the two tangents have the same length, PA = PB. For this specific case, x2 + 10 = 46, which yields x2 = 36. Therefore, the value of x is 6. In a related problem, it's known that the angle subtended by a diameter at any point on the circumference is equal to 90 degrees. Additionally, angles in a cyclic quadrilateral are supplementary, meaning they add up to 180 degrees. The opposite angles in a cyclic quadrilateral are also equal. Furthermore, a circle theorem states that angles formed in the same segment of a circle are always equal. Circle theorems are important because they help solve geometry problems and provide useful patterns and theorems for both practical and theoretical situations. To find angles using circle theorems, one can apply various theorems based on given conditions and shapes. Circle theorems are fundamental concepts in geometry that help solve problems related to circles. These statements describe essential properties of circle components such as chords, radius, center, circumference, tangent, segment, sector, and arc. We'll explore various theorems and their proofs, applying them to examples to understand their practical uses. Let's begin by defining these key terms: Chord - a line segment touching the circle at two points; Radius - the fixed distance between the center and any point on the boundary; Center - the fixed point equidistant from all circle points; Circumference - the perimeter of the circle; Tangent - a line segment touching the circle at one unique point; Segment - the area enclosed by a chord and arc; Sector - the area enclosed by radii and an arc; Arc - a curve portion of the circumference. Some essential circle theorems include: 1. The angle subtended by a chord at the center is twice the angle subtended by it at the circumference. 2. The angle subtended by the diameter at the circumference is a right angle. 3. The angles subtended at the circumference by the same arc are equal. 4. Two equal chords subtend equal angles at the circle's center. 5. If the angles subtended by two chords at the center are equal, then the two chords are equal. 6. The opposite angles in a cyclic quadrilateral are supplementary. 7. The angle between the radius and the tangent at the point of contact is 90 degrees. These theorems will be proven and applied to examples, illustrating their practical applications in geometry. ∠OBC = ∠OCB because OB = OC (radii), so angles opposite equal sides are equal, therefore ∠AOD=2×∠ACO ⋯ (1) ∠DOB=2×∠OCB ⋯ (2) Adding equations (1) and (2), we get ∠AOB=2×(∠ACO+∠OCB) ⇒ ∠AOB=2×∠ACB Hence, we proved the circle theorem 'The angle subtended by a chord at the center is twice the angle subtended by it at the circumference.' We also have another theorem: The angle subtended by the diameter at the circumference is a right angle. We can prove this using the first theorem and the fact that AB is a straight line (diameter). ∠AOB = 180° as AB is a straight line, so 2×∠ACB = 180° which implies ∠ACB = 90°. Another theorem states: The angles subtended at the circumference by the same arc are equal. We can prove this using the circle theorem 'The angle subtended by a chord at the center is twice the angle subtended by it at the circumference.', and we get ∠ACB=∠ADB. Two equal chords subtend equal angles at the center of the circle is another theorem, which can be proved by showing that triangles AOB and COD are congruent. Lastly, if the angles subtended by two chords at the center are equal, then the two chords are equal. This can also be proved using congruent triangles. Given text here: Sine OS and OT are radii, OS = OT. Using the circle theorem 'The angle between the radius and the tangent at the point of contact is 90 degrees.', we have ∠OTP = 90°. In triangle OTP, using angle sum theorem, we have ∠TOP + ∠OTP + ∠OPT = 180° ⇒ ∠TOP + 90° + 32° = 180° ⇒ ∠TOP = 180° - (90° + 32°) = 58° Since, OS = OT ⇒ ∠OSP = ∠OTP = x (because angles opposite to equal sides are equal). Using the exterior angle theorem, we have ∠OSP + ∠OTP = ∠TOP ⇒ x + x = 58° ⇒ 2x = 58° ⇒ x = 29° Answer: x = 29° Example 2: Consider the circle given below with center O. Find the angle x using the circle theorems. Solution: Using the circle theorem 'The angle subtended by the diameter at the circumference is a right angle.', we have ∠ABC = 90°. So, using the triangle sum theorem, ∠BAC + ∠ACB + ∠ABC = 180° ⇒ x + 55° + 90° = 180° ⇒ x + 145° = 180° ⇒ x = 180° - 145° = 35° Answer: x = 35° Example 3: Consider the circle given below with center O. Find the value of y using the circle theorems. Solution: To find the value of y, we will use the circle theorem 'The angle subtended by a chord at the center is twice the angle subtended by it at the circumference. '. So, ∠POR = 2∠PQR We are given ∠PQR = 50°, so we have ∠POR = 2∠PQR ⇒ y = 2 × 50° = 100° Answer: y = 100° View Answer > go to slidego to slidego to slide Want to build a strong foundation in Math? Go beyond memorizing formulas and understand the ‘why’ behind them. Experience Cuemath and get started. Book a Free Trial Class FAQs on Circle Theorems Circle theorems are statements in geometry that state important results related to circles. These theorems state important facts about different components of a circle such as a chord, segments, sector, diameter, tangent, etc. What Does Subtend Mean in Circle Theorems? Subtend in circle theorems means an angle that is subtended opposite an arc or a chord. When we say that an angle is subtended by an arc, this implies that the angle is made by the lines from the arc on the opposite side of the circle. Why Do We Need Circle Theorems? We need circle theorems to solve various problems in geometry. When we draw angles and lines inside a circle, we can deduce various patterns and theorems from it which are helpful in practical and theoretical life. How to Find Angles in Circle Theorems? We can find angles in circles using the circle theorems based on angles. Some of the important circle theorems based on angles are: The angle subtended by a chord at the center is twice the angle subtended by it at the circumference. The angle subtended by the diameter at the circumference is a right angle. The angles subtended at the circumference by the same arc are equal. List Circle Theorems Statements. The circle theorems are statements that state results about various components of circle. Some of the important circle theorems statements are: The angle subtended by a chord at the center is twice the angle subtended by it at the circumference. The angle subtended by the diameter at the circumference is a right angle. The angles subtended at the circumference by the same arc are equal. Two equal chords subtend equal angles at the center of the circle. If the Paraphrased text: A key property is that if two chords intersect at the centre of a circle and their lengths are equal, then those chords themselves will be equivalent. In cyclic quadrilaterals, pairs of opposite angles always add up to 180 degrees. Furthermore, the angle formed by the radius and the tangent line at the point where they touch is a right-angled triangle with a 90-degree measure. To validate circle theorems, we leverage foundational concepts in geometry such as the triangle sum theorem, other established circle principles, and rules governing angular relationships within geometric shapes. Circle theorems are integral components of geometry because a circle itself is a two-dimensional plane figure that significantly contributes to the study of geometry. As such, all related theorems pertaining to circles are essential elements in understanding the broader realm of geometry.